Predictions for the Ooty Wide Field Array (OWFA)

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OWFA (Ooty Wide Field Array)

dipoles

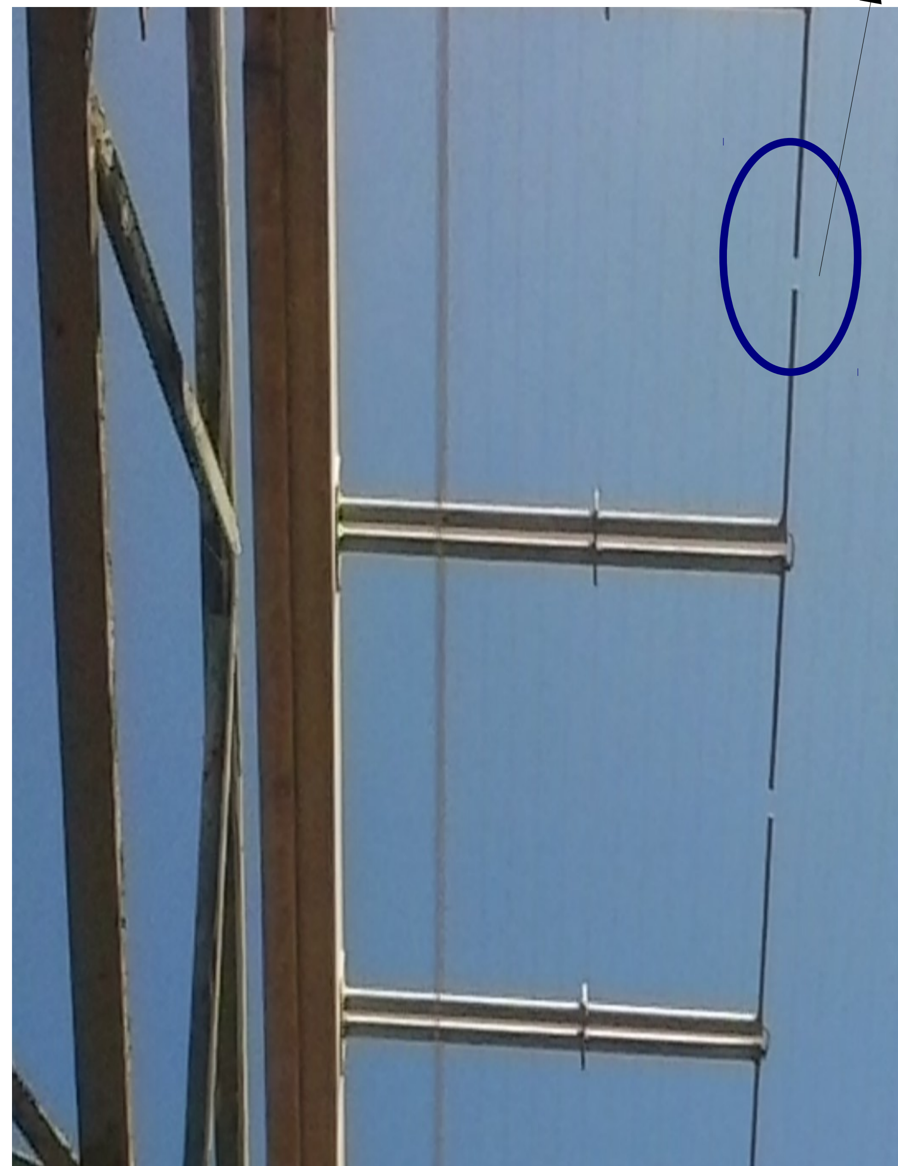


Photo Courtesy: Siddhartha Bhattacharya

OWFA is a upgraded 530 m long and 30 m wide parabolic cyllindrical reflector, to function as a linear radio-interferrometric array.

Total of 1056 dipoles, arranged from end to end along the axis of the cyllinder

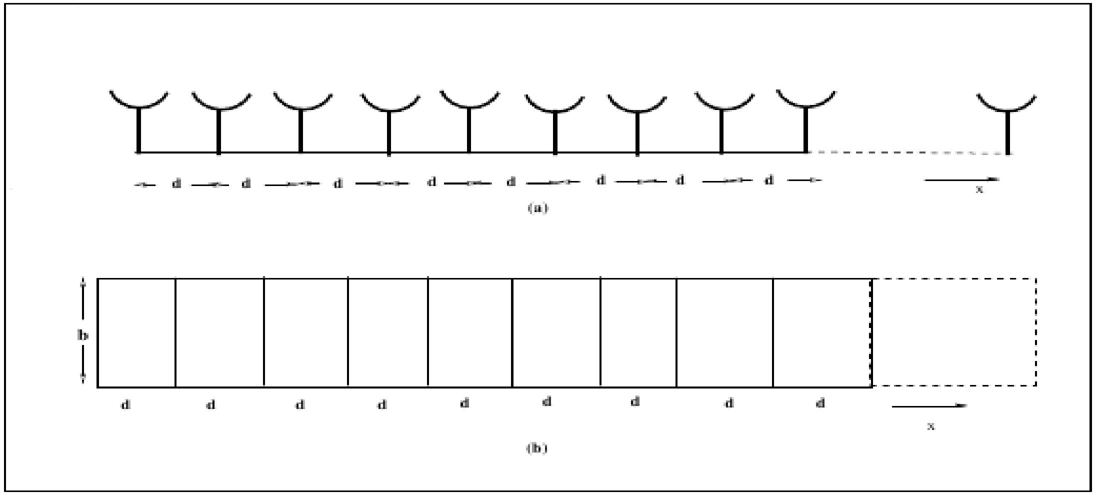
Operates at a central frequency of 326.5 Mhz, which directly correspond to measuring the HI radiation from redshift z = 3.35

Parameter	Phase I	Phase II
No. of antennas (N_A)	40	264
Aperture dimensions ($b \times d$)	$30\,\mathrm{m} \times 11.5\,\mathrm{m}$	$30\,\mathrm{m} \times 1.92\,\mathrm{m}$
Field of View(FoV)	$1.75^\circ \times 4.6^\circ$	$1.75^\circ \times 27.4^\circ$
Smallest baseline (d_{min})	$11.5\,\mathrm{m}$	$1.9\,\mathrm{m}$
Largest baseline (d_{max})	$448.5\,\mathrm{m}$	$505.0\,\mathrm{m}$
Angular resolution	$7'$	$6.3'$
Total bandwidth (B)	18 MHz	30 MHz
Single Visibility rms. noise (σ) assuming $T_{sys} = 150\,\mathrm{K}, \eta = 0.6,$ $\Delta\nu_c = 0.1\,\mathrm{MHz}, \Delta t = 16\,\mathrm{s}$	1.12 Jy	6.69 Jy

Courtesy: Ali, S. S., Bharadwaj, S. 2014, JOAA, 35, 157

$$\nu_c = 326.5\,\mathrm{MHz} \longrightarrow \text{Redshift, } z = 3.35$$

Post-reionization era $(z \leq 6)$



Baseline configuration of OWFA

$$\mathbf{U}_1 = \left(\frac{d}{\lambda}\right)\hat{i}; \quad \mathbf{U}_2 = 2\mathbf{U}_1; \quad \mathbf{U}_3 = 3\mathbf{U}_1; \ldots \quad \mathbf{U}_{N_A-1} = (N_A - 1)\mathbf{U}_1$$

$$\vec{U}_n = n\left(\frac{d\nu}{c}\right)\hat{i}$$

$$A(\vec{\theta}, \nu) = \mathrm{sinc}^2\left(\frac{\pi d \theta_x}{\lambda}\right)\mathrm{sinc}^2\left(\frac{\pi b \theta_y}{\lambda}\right)$$

$$(N_A - n)$$

Primary beam pattern of OWFA

Redundant baselines

Courtesy: Ali, S. S., Bharadwaj, S. 2014, JOAA, 35, 157

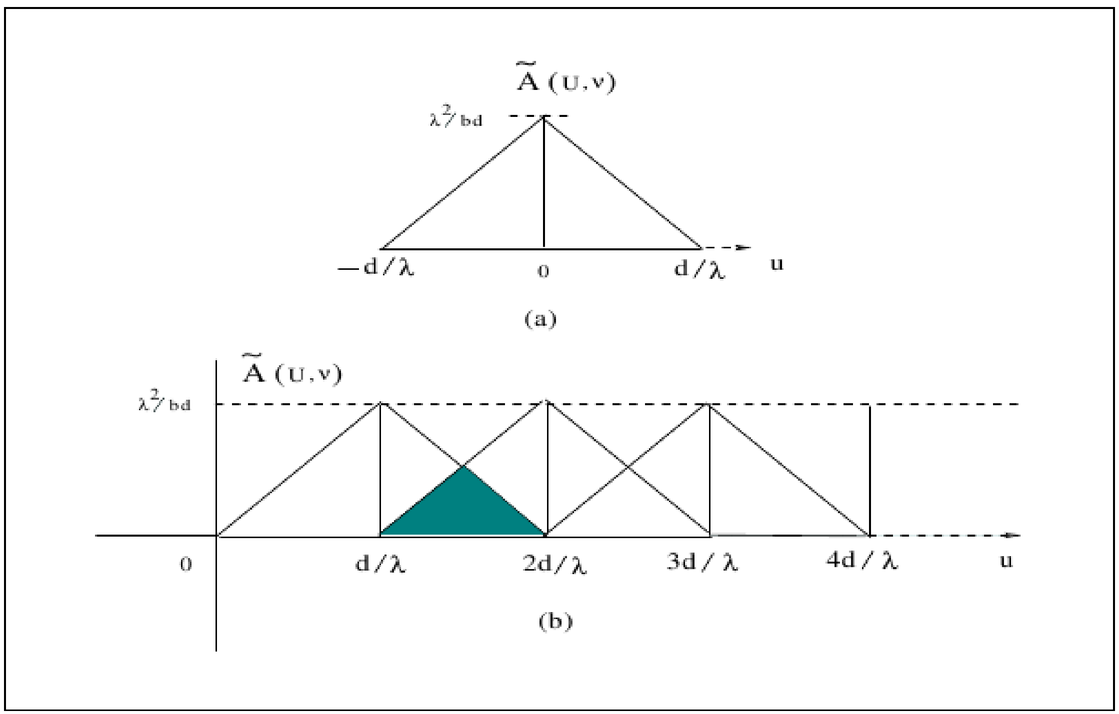
Signal Visibility

$$\mathcal{V}(\vec{U},\nu) = \int \delta I(\vec{\theta},\nu)\, A(\vec{\theta},\nu)\, e^{2\pi i \vec{U}.\vec{\theta}}\, d^2\theta$$

In Fourier space

$$\int \tilde{A}(\vec{U}-\vec{U}',\nu)\Delta\tilde{I}(\vec{U}',\nu)d^2\vec{U}'$$

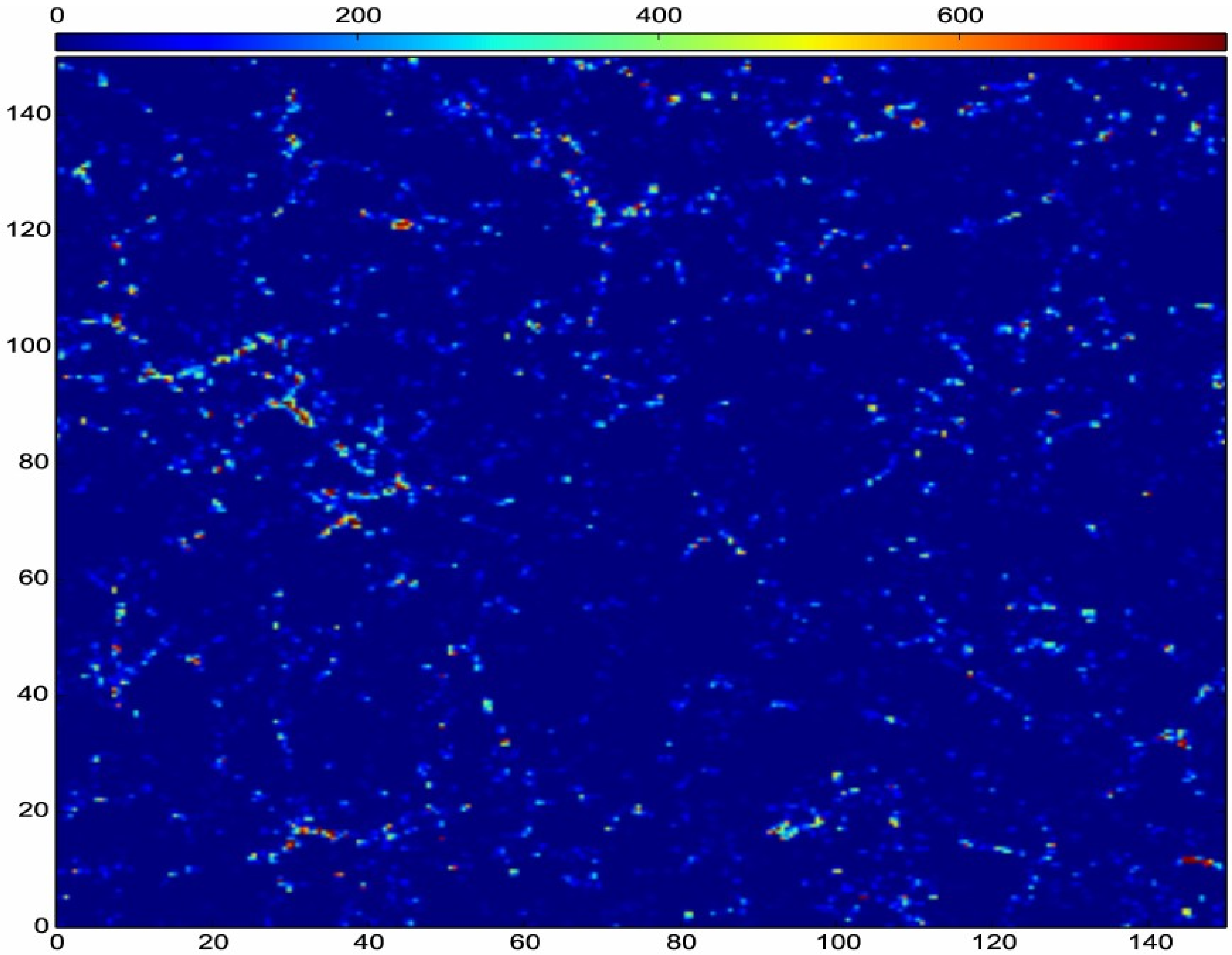
$$\vec{U}_n - \frac{d\nu}{c} \le \vec{U}' \le \vec{U}_n + \frac{d\nu}{c}$$



Aperture power pattern of antenna

Courtesy: Ali, S. S., Bharadwaj, S. 2014, JOAA, 35, 157

HI distribution in the post reionization era, z = 3.35



Courtesy: Sarkar, Debanjan et al., MNRAS, 2016

Bulk of the neutral hydrogen (HI) in the post reionization era resides in the dense pockets of the self-shielded HI, which are indentified as the Damped Lyman Alpha Syatems (DLA's) in quasar obsertvations.

Fluctuations in the combined emission of the neutral hydrogen holds the signarture of the underlying source clustering at that epoch.

$$\boxed{v(\vec{U},\tau_m) = \Delta\nu_c \sum_{n=1}^{N_c} \mathcal{V}(\vec{U},\nu)\, e^{2\pi i \tau_m \nu_n}}$$

delay channels

Total number of frequency channels

$$\boxed{\tau_m = \frac{m}{B}}$$

$$\boxed{-N_c/2 \leq m < N_c/2}$$

$$\boxed{C_{ab}(m) = \langle v(\vec{U}_a,\tau_m)\, v^*(\vec{U}_b,\tau_m)\rangle}$$

Assumptions

Visibility consists of HI signal and Noise

Forgerounds have been ignored

$$C_{ab}(m) = \frac{B}{r_\nu^2 r'_\nu}\left(\frac{2k_B}{\lambda^2}\right)^2 \int d^2U'\, \tilde{A}(\mathbf{U}_a - \mathbf{U}')\tilde{A}^*(\mathbf{U}_b - \mathbf{U}')P_{\rm HI}\left(\frac{2\pi\mathbf{U}'}{r_\nu}, \frac{2\pi\tau_m}{r'_\nu}\right) + \delta_{a,b}\, 2\,\Delta\nu_c\, B\, \frac{\sigma^2}{(N_A - a)}\,. \qquad (7)$$

HI signal part

Noise part

$$\vec{k}_\perp = \frac{2\pi\vec{U}}{r_\nu} \qquad k_{\|m} = \frac{2\pi\tau_m}{r'_\nu}$$

$$k = \sqrt{k_\perp^2 + k_\|^2}$$

Neutral HI fraction

HI bias

Redshift distortion parameter

Amplitude of the HI power spectrum

$$A_{HI} = \bar{x}_{HI}\, b_{HI}$$

$$\vec{k} = (\vec{k}_\perp, k_\|)$$

$$k_\|/k$$

$$P_{HI}(\vec{k}) = \bar{x}^2_{HI}\, b^2_{HI}\,(1+\beta\mu^2)^2 \left(\frac{D(z)}{D(0)}\right)^2 P(k)$$

ΛCDM **Cosmological parameters**

Matter power spectrum

Inverse of the error covariance matrix

Fisher Matrix →

$$\boxed{F_{\alpha\gamma} = \frac{1}{2}\sum_m C^{-1}_{ab}(m)\,[C_{bc}]_{,\alpha}\,C^{-1}_{cd}(m)\,[C_{da}]_{,\gamma}}$$

$$\boxed{(\alpha,\gamma) = (A_{HI},\beta)}$$

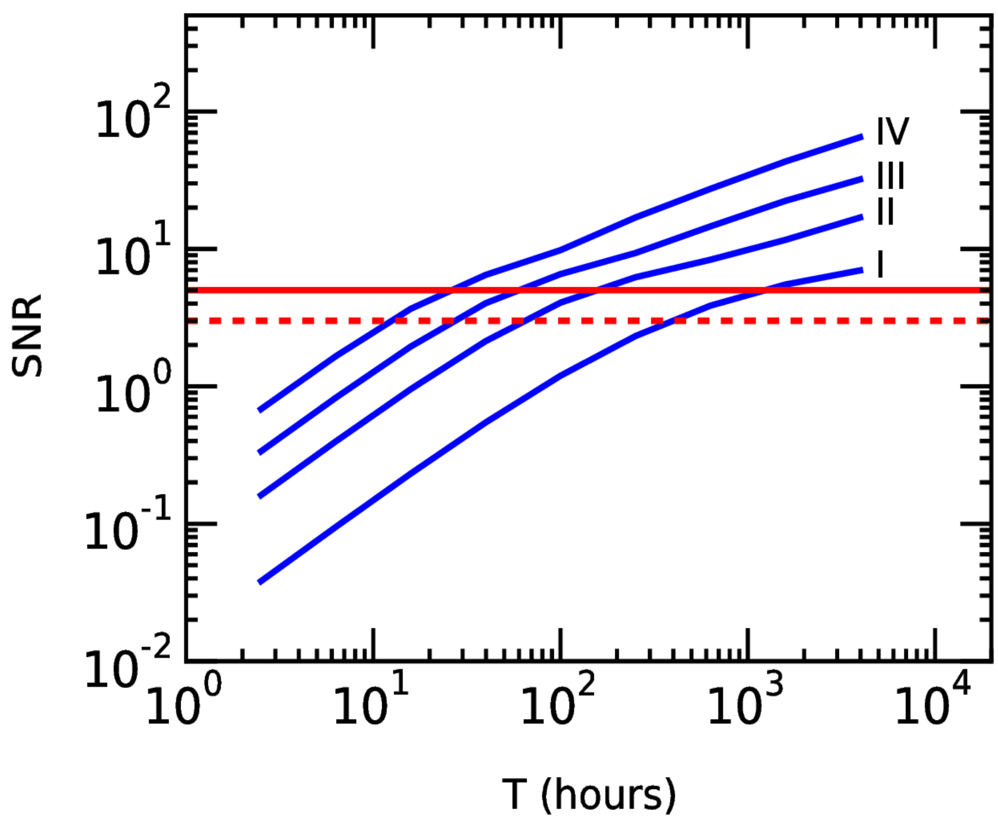
→ **Parameters for the Fisher matrix analysis**

$$\boxed{A_{HI} = 4.93\times 10^{-2}}$$

$$\boxed{\mathrm{SNR} = \frac{A_{HI}}{\Delta A_{HI}}}$$

Fiducial value

Marginalized



Prior →

$$\boxed{0.33 \le \beta \le 1}$$

Phase	SNR	t_C (hr)	t_M (hr)
Phase I	5, 3	800, 350	1190, 390
Phase II	5, 3	110, 60	150, 70
Phase III	5, 3	50, 20	50, 20
Phase IV	5, 3	20, 10	25, 15

Binned HI power spectrum estimates using OWFA

$$\boxed{P^{s}_{HI}(\vec{k}) = (1+\beta\mu^{2})^{2}\, P^{r}_{HI}(k)}$$

Real space HI power spectrum

Redshift space HI power spectrum

Outline of the analysis

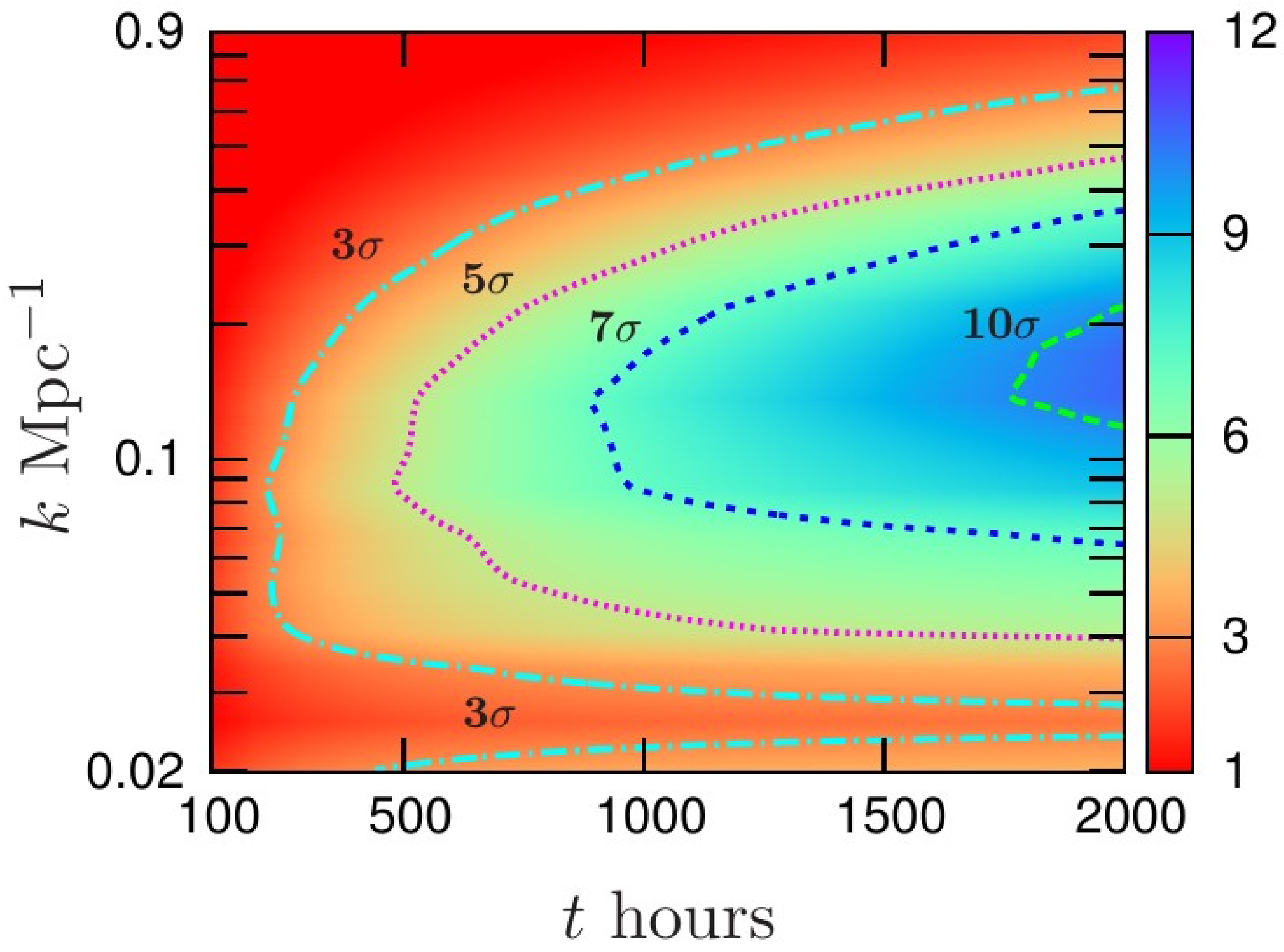
$$0.018 \leq k \leq 2.73\,\mathrm{Mpc}^{-1}$$

OWFA Phase II k-range

Divide into 20 equispaced logarithmic k-bins

Average value of the HI power spectrum is chosen to be the fiducial value in each k-bin, total of 20 parameters

Fisher matrix with 20 parameters



$$\boxed{\mathrm{SNR} = \frac{[P^r_{HI}]^i}{\Delta[P^r_{HI}]^i}}$$

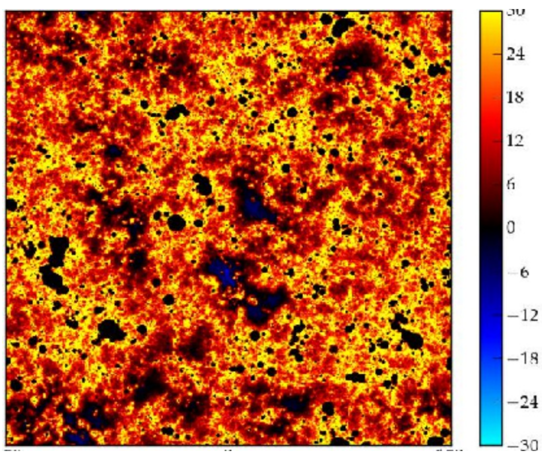
Fiducial value at i-th k-bin

$0.05 \leq k \leq 0.3\,\mathrm{Mpc}^{-1}$ + $1000\,\mathrm{hrs}$ $\longrightarrow$ **Detection** $\sim 5\sigma$

To generate visibilities analytically for a given instrument

Visibility → **HI signal contribution to the visibility**

$$\boxed{\mathcal{V}(\vec{U},\nu) = \int \delta I(\vec{\theta},\nu)\, A(\vec{\theta},\nu)\, e^{2\pi i \vec{U}.\vec{\theta}}\, d^2\theta}$$



$\delta T_b(\vec{\theta},\nu)$

multiply

$2k_B/\lambda^2 A(\vec{\theta},\nu)e^{2\pi i \vec{U}.\vec{\theta}}$

integrate

$\mathcal{V}(\vec{U},\nu)$

$$\delta I(\vec{\theta},\nu) = \left(\frac{\partial B}{\partial T}\right)\delta T_b(\vec{\theta},\nu)$$

$2k_B\nu^2/c^2$

Signal covariance: ergodic case

$$S_2(\vec{U}, \nu_1, \nu_2) = \langle \mathcal{S}(\vec{U}, \nu_1) \mathcal{S}^*(\vec{U}, \nu_2) \rangle$$

$$\left(\frac{\partial B}{\partial T}\right)^2_{\nu_c} \int \frac{d^3k}{(2\pi)^3} \left| \tilde{a}(\vec{U} - \frac{\vec{k}_\perp r_c}{2\pi}) \right|^2_{\nu_c} P_T(\vec{k})\, e^{i r_c' k_\parallel (\nu_2 - \nu_1)}$$

HI brightness temperature power spectrum

$$S_2(\vec{U}, \nu_1, \nu_2) = S_2(\vec{U}, \Delta\nu)$$

Ergodic in frequency

$$|\nu_2 - \nu_1|$$

Breaking the ergodicity.............

$$\mathbf{S}(\vec{u}_a, \nu_a; \vec{u}_b, \nu_b) = \frac{1}{(2\pi)^3} \left(\frac{2k_B\nu_a^2}{c^2}\right)\left(\frac{2k_B\nu_b^2}{c^2}\right)]$$

$$\int_{-\infty}^{\infty} \tilde{a}\left(\frac{\nu_a \vec{u}_a}{\nu_c} - \frac{r_{\nu_a}\vec{k}_\perp}{2\pi}\right) \tilde{a}^*\left(\frac{\nu_b \vec{u}_b}{\nu_c} - \frac{r_{\nu_b}\vec{k}'_\perp}{2\pi}\right)$$

$$\left[(1+\beta_a \mu^2)\bar{x}_{HI}(z_a) b_{HI}(z_a)\bar{T}(z_a)\left(\frac{D(z_a)}{D(0)}\right)\right]$$

$$\left[(1+\beta_b \mu^2)\bar{x}_{HI}(z_b) b_{HI}(z_b)\bar{T} \quad \vec{u} = n\left(\frac{d\nu_c}{c}\right)\hat{i}\right.$$

$$P(\vec{k}_\perp, k_\parallel)\mathrm{sinc}(k_\parallel r'_{\nu_a}\Delta\nu/2)\mathrm{sinc}(k_\parallel r'_{\nu_b}\Delta\nu/2)$$

$$\cos(k_\parallel[r_{\nu_a} - r_{\nu_b}])\ d^3k.$$

$$\vec{u}_n = n\left(\frac{d\nu_c}{c}\right)\hat{i}$$

Baselines are defined with respect to central frequency

$$\bar{x}_{HI}(z), b_{HI}(z), \bar{T}(z)$$

$$r_\nu, r'_\nu$$

$$\nu = \frac{1420}{1+z}$$

$$S_2(\vec{u}_a, \nu_a; \vec{u}_b, \nu_b) \neq S_2(\vec{u}_a, \vec{u}_b, |\nu_b - \nu_a|)$$

Signal Co-variance

$$V_2(\vec{u}_1, \nu_1; \vec{u}_2, \nu_2)$$

$\Big\downarrow$ Eigendecompose

Eigenvectors and Eigenvalues

$$\lambda_k, \hat{e}_{ik}$$

$\Big\downarrow$ use set of Gaussian random numbers

Visibilities

$$\mathcal{V}(\vec{u}, \nu)$$

Realizations of Visibilities

$$\mathcal{V}_i = \sum_{k=1}^{N} \sqrt{\frac{\lambda_k}{2}} (a_k + i b_k) \hat{e}_{ik}$$

<u>**KKL-decomposition of visibility**</u>

$$\mathcal{V}_i = \sum_{k} Z_k \phi_i^k$$

Single realization of visibility

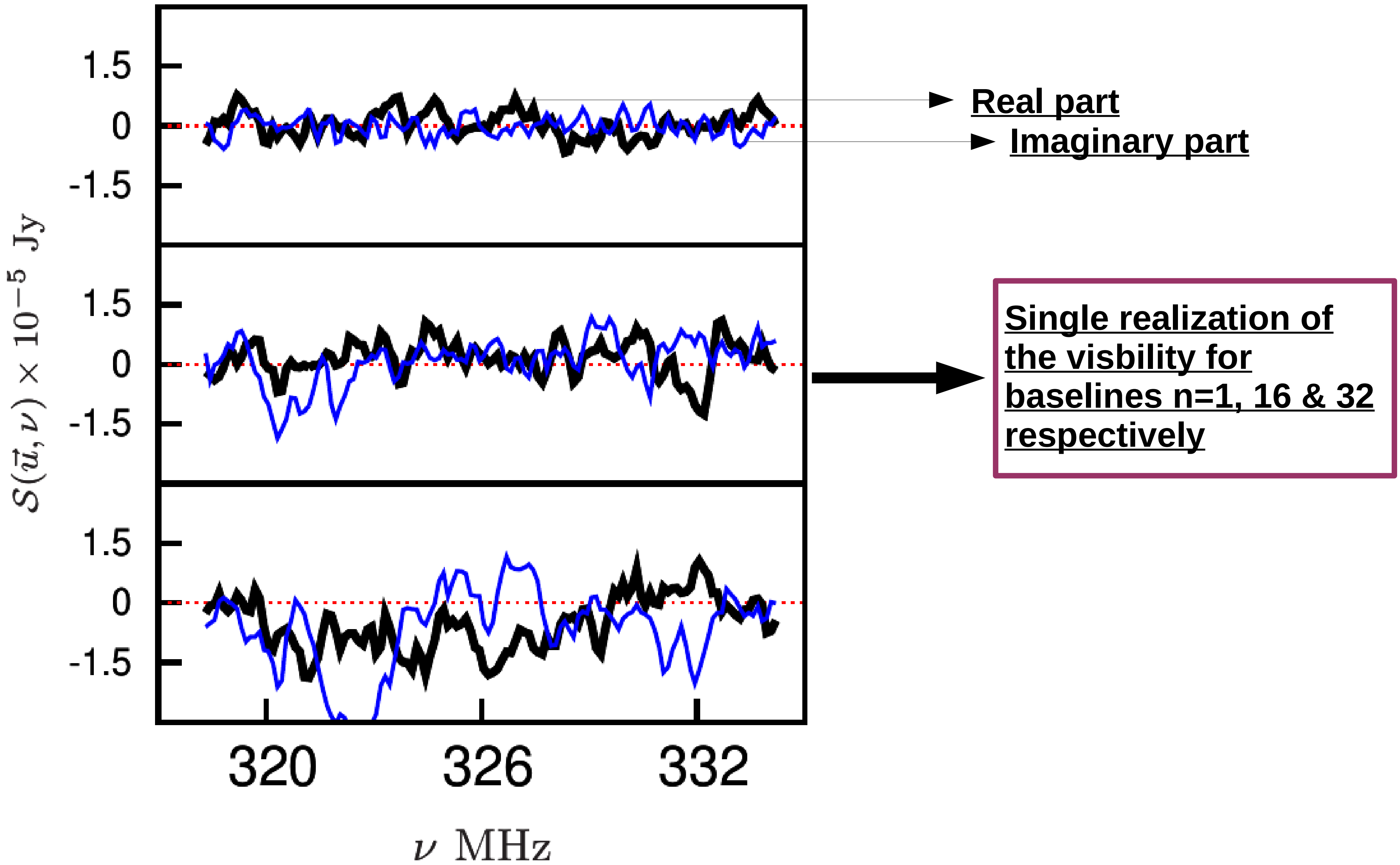
Uncorrelated Gaussian random numbers having zero mean & unit variance

k-th eigenvalue of the signal covariance matrix

Eigenvector corresponding to the k-th eigenvalue

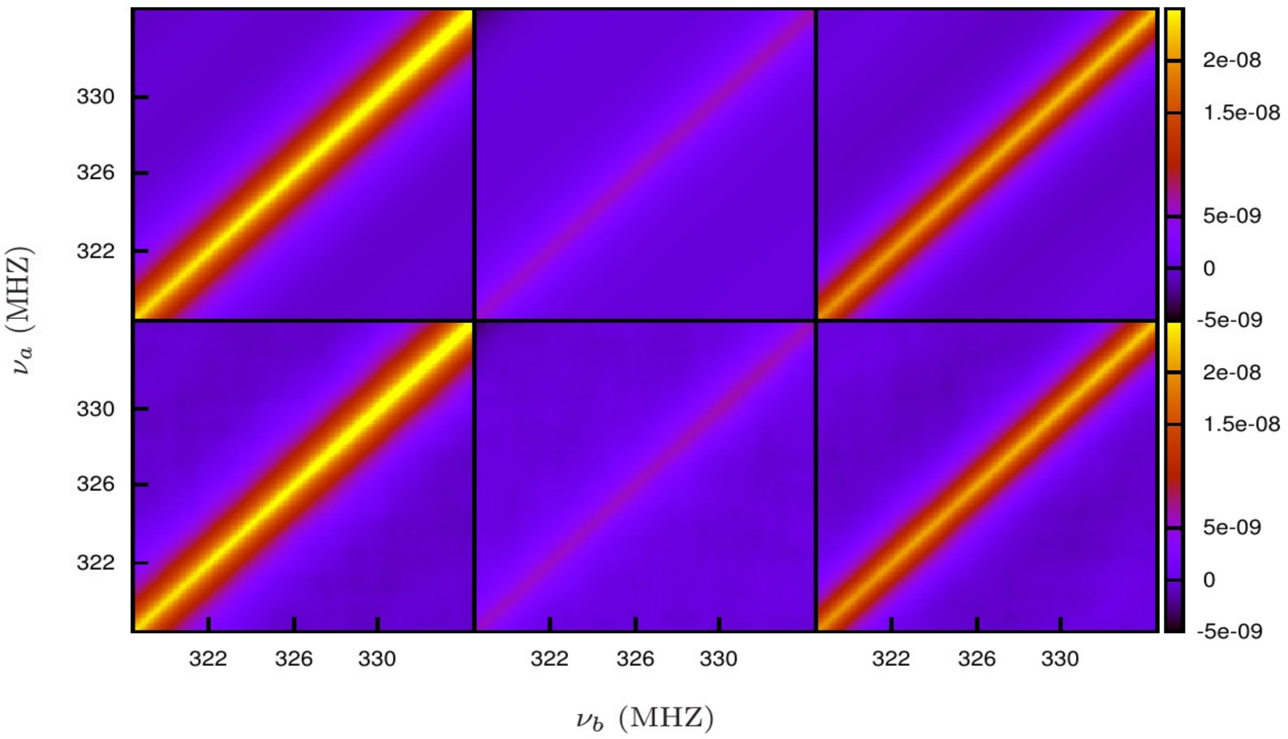
Number of eigenvalues or the order of the signal covariance matrix

Assumption → <u>**Visibilities are assumed to be a Gaussian random field**</u>



1.5
0
-1.5
1.5
0
-1.5
1.5
0
-1.5
320
326
332
ν MHz
S(u⃗, ν) × 10^−5 Jy
Real part
Imaginary part
Single realization of the visbility for baselines n=1, 16 & 32 respectively

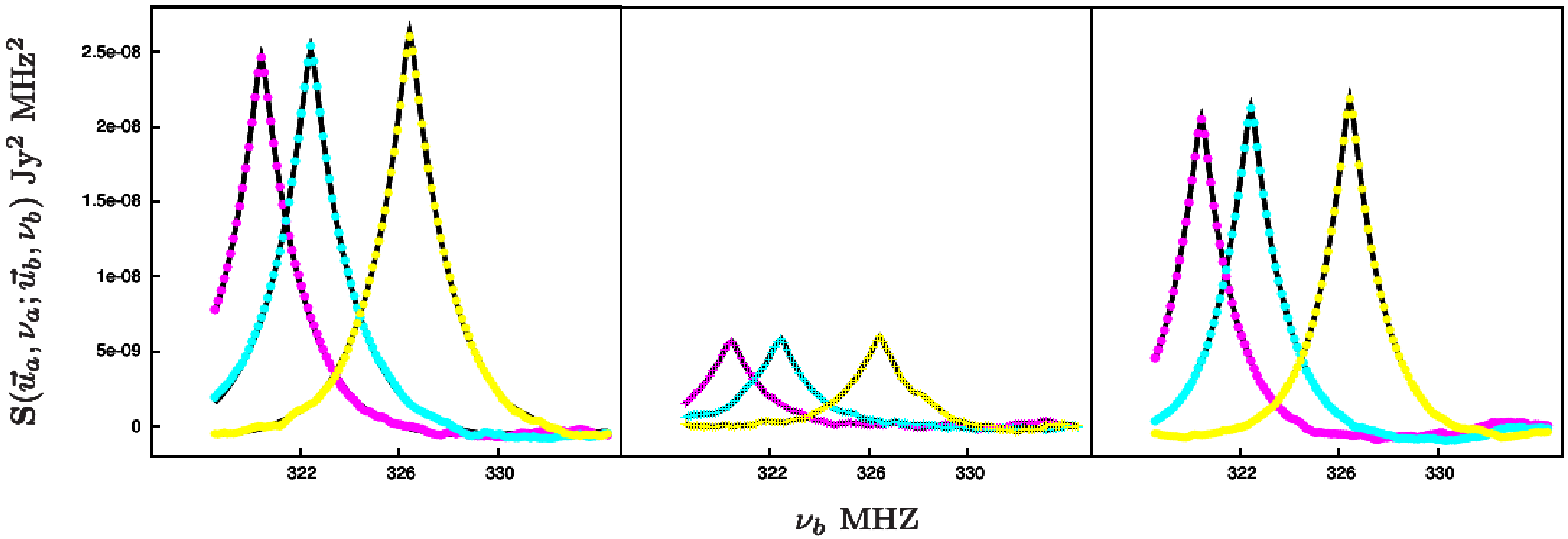
Results: signal covariance



Theoretical

Reconstructed

(No. of realizations = 10000)



Summary

1. We have eigendecomposed the expected theoretical signal covariance matrix for OWFA, eigenvectors of which give the orthonormal basis or the Koshambi-Karhunen-Loeve (KKL)-basis of the HI signal visibilities.

2. Using the eigenvalues and the eigenvectors, we have generated different Gaussian random realizations of the visibilities.

3. The method described here has the advantage of incorporating the light-cone effect.

Working group of OWFA